On the use of Machine Learning methods to estimate Triangular Two-level Panel Data Models with Individual Fixed Effects

Authors: Monika Avila Márquez, Jaya Krishnakumar

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MAD, University of Geneva
GSEM, University of Geneva

Motivation

Motivation

- Linear structural equation for individual i:

$$y_{it} = x_{it1}\beta_1 + \tilde{x}_{it}'\beta_2 + \alpha_i + \varepsilon_{it}, \quad t = 1, 2, ..., T, \qquad (1)$$

  where y_{it} outcome variable, x_{it1} endogenous explanatory variable (EEV), $\tilde{x}_{it}$ vector of exogenous regressors, α_i individual fixed effect.
- EEV has a nonlinear relationship with the instrumental variables and the exogenous regressors:

$$x_{it1} = g(\tilde{x}_{it}, z_{it}) + \alpha_i + u_{it}, \quad t = 1, 2, ..., T, \quad i = 1, 2, ..., N, \qquad (2)$$

  with z_{it} instrumental variables,
  $E[u_{it}|\tilde{x}_{i1}, z_{i1}, ..., \tilde{x}_{iT}, z_{iT}, \alpha_i] = 0$.
- Parameter of interest: β_1.

Motivation

Source of Identification: Exogenous variation (Instrumental variables).

But..

- Should we perform 2SLS on the transformed model ignoring the nonlinearity in the reduced form equation?
- Would be beneficial to take into account the nonlinearity in the reduced form equation?
- Wooldridge (2015), CF is more efficient than 2SLS when the nonlinear reduced form equation is correctly specified but it is non robust to misspecification of the nonlinear form.

This paper

Control function approach using random forest regression to estimate the nonlinear relationship.

Main contributions and results

1. Linear structural equation:
   - 1.1 Control function approach.
   - 1.2 Two estimation methods (CF, CF-OS) using random forest regression to estimate the nonlinear relationship.
   - 1.3 Using a Monte Carlo experiment, we conclude that CF-OS has lower bias and RMSE than 2SLS on the first-differenced model in almost all scenarios.
2. Partial linear structural equation:
   - 2.1 Control function approach.
   - 2.2 Three methods (CF, CF-OS, CF-IT) using random forest regression to estimate the nonlinear relationship.
   - 2.3 Using a Monte Carlo experiment, we conclude that CF-IT with data splitting has lower bias and RMSE than 2SLS on the first-differenced model.

Outline

1. Literature review
2. The setting: linear structural equation.
   - 2.1 Identification
   - 2.2 Estimation
   - 2.3 Monte Carlo experiment
3. Relaxing the linearity assumption in the structural equation.
   - 3.1 Identification
   - 3.2 Estimation
   - 3.3 Monte Carlo experiment
4. Conclusions

Literature Review

1. Control function approach:
   1.1 Wooldridge (2015), CF is more efficient than 2SLS when the nonlinear reduced form equation is correctly specified.
2. Semiparametric linear panel data model with EEV.
   2.1 RE: Li and Stengos (1996), IV technique using kernel regression and keeping the model in levels; FE: Qiang and Wang (2012), IV technique using kernel regression on the first-differenced model; Baltagi and Li (2002), IV technique using series estimator for the first-differenced model.

The set up: Linear structural equation

The set up

A.1 Observed data: $w_i = (y_i, x_{i1}, \tilde{x}_i, z_i)$ are $T \times 1$ identical and independent copies of the vector random variables $(\mathbf{y}, \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_K, \mathbf{z})$. $N \to \infty$, T fixed and small.

A.2a Linear structural equation:

$$y_{it} = x_{it1}\beta_1 + \tilde{x}'_{it}\beta_2 + \alpha_i + \varepsilon_{it}, \quad t = 1, 2, ..., T, \quad i = 1, 2, ..., N. \tag{3}$$

A.3 α_i individual fixed effect.

A.4 $\tilde{x}_{it}$ is strictly exogeneous

$$E[\varepsilon_{it} | \tilde{x}_{i1}, \tilde{x}_{i2}, ..., \tilde{x}_{iT}, \alpha_i] = 0. \tag{4}$$

Linear structural equation

A.5 x_{it1} is an endogenous explanatory variable (EEV). It is nonlinearly related to $\tilde{x}_{it}$, and the instrumental variables z_{it}.

$$x_{it1} = g(\tilde{x}_{it}, z_{it}) + \alpha_i + u_{it}, \quad t = 1, 2, ..., T, \quad i = 1, 2, ..., N, \quad (5)$$

with $g(.)$ a smooth function,
$E[u_{it}|\tilde{x}_{i1}, \tilde{x}_{i2}, ..., \tilde{x}_{iT}, z_{i1}, z_{i2}, ..., z_{iT}, \alpha_i] = 0,,$
$E[\varepsilon_{it}|z_{i1}, z_{i2}, ..., z_{iT}, \alpha_i] = 0.$
A.6 The structural and reduced-form errors are linearly dependent

$$\varepsilon_{it} = \rho u_{it} + \omega_{it}, \quad t = 1, 2, ..., T, \quad i = 1, 2, ..., N, \quad (6)$$

with $E[\omega_{it}|u_{i1}, u_{i2}, ..., u_{iT}] = 0.$

Identification: Control Function approach

1. Augment the model using $\varepsilon_{it} = \rho u_{it} + \omega_{it}$, and obtain:

$$y_{it} = x_{it1}\beta_1 + \tilde{x}'_{it}\beta_2 + \alpha_i + \rho u_{it} + \omega_{it}, \quad t = 1, 2, ..., T, \quad i = 1, 2, ..., N. \tag{7}$$

2. First-difference the model to eliminate the individual fixed effect:

$$\Delta y_{it} = \Delta x_{it1}\beta_1 + \Delta \tilde{x}'_{it}\beta_2 + \rho \Delta u_{it} + \Delta \omega_{it}. \tag{8}$$

   The population moment conditions are:

$$\mathbb{E}[\Delta \omega_{it} \Delta H_{it}] = 0 \quad \forall t, \tag{9}$$

   with $\Delta H_{it} = [\Delta x_{it1} \quad \Delta \tilde{x}'_{it} \quad \Delta u_{it}]'$.

3. Identification of $\Delta g(\tilde{x}_{it}, z_{it})$ is guaranteed under assumption A.5.

Estimation: Method 1

Step 1: Estimate $\Delta g(\tilde{x}_{it}, z_{it})$ by running a random forest regression of Δx_{it1} on $\tilde{x}_{it}, \tilde{x}_{it-1}, z_{it}, z_{it-1}$, and obtain $\widehat{\Delta u_{it}} = \Delta x_{1it} - \widehat{\Delta g(\tilde{x}_{it}, z_{it})}$.

Step 2: Use the estimated residuals $\widehat{\Delta u_{it}}$ in place of Δu_{it} in the first-differenced augmented model:

$$\Delta y_{it} = \Delta x_{it1}\beta_1 + \Delta \tilde{x}_{it}'\beta_2 + \rho \widehat{\Delta u_{it}} + \Delta \omega_{it}. \tag{10}$$

Estimate the parameters of the augmented model using the sample moment conditions:

$$\frac{1}{N(T-1)} \sum_{i=1}^{N} \sum_{t=2}^{T} \Delta \omega_{it} \widehat{\Delta H_{it}} = 0. \tag{11}$$

Estimation: Method 2

Step 1: Estimate $\Delta g(\tilde{x}_{it}, z_{it})$ by running a random forest regression of Δx_{it1} on $\tilde{x}_{it}, \tilde{x}_{it-1}, z_{it}, z_{it-1}$, and obtain $\widehat{\Delta u_{it}} = \Delta x_{it1} - \widehat{\Delta g(\tilde{x}_{it}, z_{it})}$.

Step 2:

2.1 Use the estimated residuals $\widehat{\Delta u_{it}}$ in place of Δu_{it} in the first-differenced augmented model:

$$\Delta y_{it} = \Delta x_{it1}\beta_1 + \Delta \tilde{x}_{it}'\beta_2 + \rho\widehat{\Delta u_{it}} + \Delta\omega_{it}. \tag{12}$$

2.2 Partial out the exogenous regressors in 12, written in matrix form:

$$M_{\Delta\tilde{x}}\Delta y_{it} = M_{\Delta\tilde{x}}\Delta x_{it1}\beta_1 + \rho M_{\Delta\tilde{x}}\widehat{\Delta u_{it}} + M_{\Delta\tilde{x}}\Delta\omega_{it}. \tag{13}$$

Estimation: Method 2

Step 2:

2.3 Estimate ρ by partialing out $M_{\Delta\tilde{x}}\Delta x_{it1}$.

2.4 Obtain $\widehat{\Delta y^*_{it}} = M_{\Delta\tilde{x}}\Delta y_{it} - \hat{\rho}_o M_{\Delta\tilde{x}}\widehat{\Delta u_{it}}$.

2.5 Estimate the parameter of interest by regressing: $\widehat{\Delta y^*_{it}}$ on $M_{\Delta\tilde{x}}\Delta x_{it1}$.

Simulation Design

$$y_{it} = \beta_1 x_{it1} + \beta_2 \tilde{x}_{it} + \alpha_i + 0.2\varepsilon_{it}^2, \tag{14}$$

where:

$$x_{it1} = g(\tilde{x}_{it}, z_{it}) + \alpha_i + 0.2u_{it}^2, \tag{15}$$

$$g(\tilde{x}_{it}, z_{it}) = \tilde{x}_{it} + z_{it} + 2exp(-16\tilde{x}_{it}^2 - 16z_{it2}^2)$$

, (16)

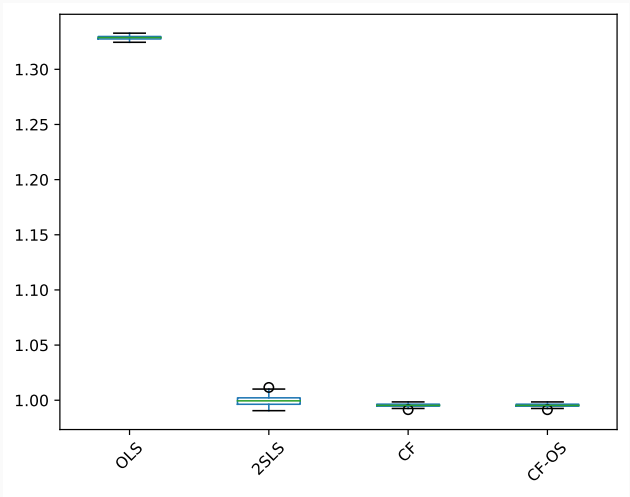
$$\tilde{x}_{it} = \alpha_i + \zeta_{it}, \quad with \quad \zeta_{it} \sim U(-2, 2), \tag{17}$$

$$z_{it} = \alpha_i + \nu_{it}, \quad with \quad \nu_{it} \sim U(-2, 2), \tag{18}$$

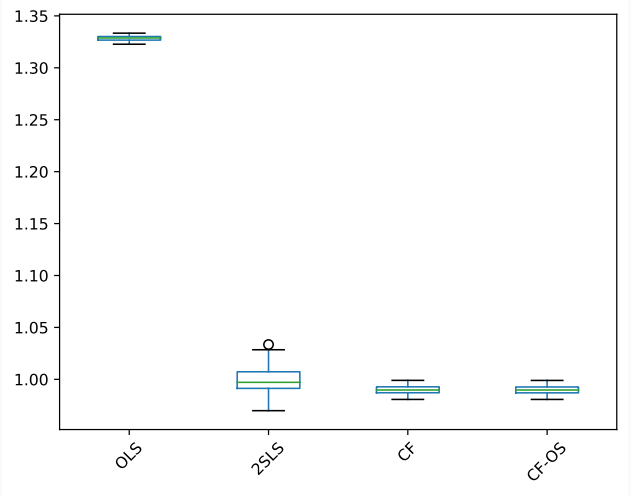
$$\begin{pmatrix} u_{it} \\ \varepsilon_{it} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0.8 \\ 0.8 & 1 \end{pmatrix} \right), \tag{19}$$

$$\alpha_i \sim N(0, 1). \tag{20}$$

Simulation results



(a) N=5000, T=20



(b) N=5000, T=3

Figure 1: Linear structural equation and nonlinear reduced-form equation: DGP 2-Scenario 1. Ivs: $\tilde{x}_{it}$, z_{it}, $\tilde{x}_{it-1}$, z_{it-1}

Note: The true parameter value is 1 ($\beta_1 = 1$).

Relaxing the linearity assumption

Relaxing the linearity assumption

A.2b The structural equation:

$$y_{it} = x_{it1}\beta_1 + h(\tilde{x}_{it}) + \alpha_i + \varepsilon_{it}. \tag{21}$$

Identification

1. Augment the model using $\varepsilon_{it} = \rho u_{it} + \omega_{it}$ to obtain:

$$y_{it} = x_{it1}\beta_1 + h(\tilde{x}_{it}) + \alpha_i + \rho u_{it} + \omega_{it}. \tag{22}$$

2. First-difference the model to eliminate the individual fixed effect:

$$\Delta y_{it} = \Delta x_{it1}\beta_1 + \Delta h(\tilde{x}_{it}) + \rho \Delta u_{it} + \Delta \omega_{it}. \tag{23}$$

3. Obtain the conditional expectation of Δy_{it} on $\tilde{x}_{it}, \tilde{x}_{it-1}$:

$$E[\Delta y_{it}|\tilde{x}_{it}, \tilde{x}_{it-1}] = E[\Delta x_{it1}|\tilde{x}_{it}, \tilde{x}_{it-1}]\beta_1 + \Delta h(\tilde{x}_{it})$$

$$+\mathrm{E}[\Delta u_{it}|\tilde{x}_{it}, \tilde{x}_{it-1}] + E[\Delta \omega_{it}|\tilde{x}_{it}, \tilde{x}_{it-1}]. \tag{24}$$

Identification

5. Obtain the deviation of Δy_{it} from the conditional expectation of Δy_{it} on $\tilde{x}_{it}, \tilde{x}_{it-1}$:

$$\Delta v_{it} = \widetilde{\Delta x_{it1}}\beta_1 + \rho\widetilde{\Delta u_{it}} + \Delta\omega_{it}. \tag{25}$$

6. Population moment conditions:
   $E[\Delta\omega_{it}\widetilde{\Delta x_{it1}}] = 0$ under A.5 and A.6.
   $E[\Delta\omega_{it}\widetilde{\Delta u_{it}}] = 0$ under A.6

Estimation: CF

The estimation procedure has three steps:

Step 1: Following Robinson1988E, estimate the reduced form residuals $\widehat{\Delta u_{it}} = \Delta x_{it1} - \widehat{E[\Delta x_{it1}|\tilde{x}_{it}, \tilde{x}_{it-1}, z_{it}, z_{it-1}]}$ using random forest regression.

Step 2: Using the residuals obtained in previous step, estimate the residuals $\widehat{\Delta v_{it}} = \Delta y_{it} - \widehat{E[\Delta y_{it}|\tilde{x}_{it}, \tilde{x}_{it-1}]}$,
$\widehat{\widetilde{\Delta x_{1it}}} = \Delta x_{it1} - \widehat{E[\Delta x_{it1}|\tilde{x}_{it}, \tilde{x}_{it-1}]}$, and
$\widehat{\widetilde{\Delta u_{it}}} = \widehat{\Delta u_{it}} - \widehat{E[\widehat{\Delta u_{it}}|\tilde{x}_{it}, \tilde{x}_{it-1}]}$ learning the conditional expectations using random forests.

Step 3: Use the estimated residuals to estimate the parameter of interest.

We call this estimator CF in the simulation study.

Estimation: CF - IT

The estimation procedure has four steps:

Step 1 as in CF.

Step 2: Set up starting values for $\rho^{(0)}$, $\beta_1^{(0)}$.

Step 3: Obtain $\Delta y_{it} - \Delta x_{it1}\beta_1^{(0)} - \rho^{(0)}\widehat{\Delta u}_{it} = \Delta h(\tilde{x}_{it}) + \Delta\omega_{it}$, and estimate $\Delta h(\tilde{x}_{it})$.

Step 4: Obtain $\Delta y_{it} - \widehat{\Delta h(\tilde{x}_{it})}$, and estimate β_1, ρ.

We call this estimator CF-IT in the simulation study.

Simulation Design

$$y_{it} = \beta_1 x_{it1} + h(\tilde{x}_{it}) + \alpha_i + 0.8u_{it} + \omega_{it}, \tag{26}$$

$$x_{it1} = g(\tilde{x}_{it}, z_{it}) + \alpha_i/2 + u_{it}, \quad with \quad u_{it} \sim N(0,1), \tag{27}$$

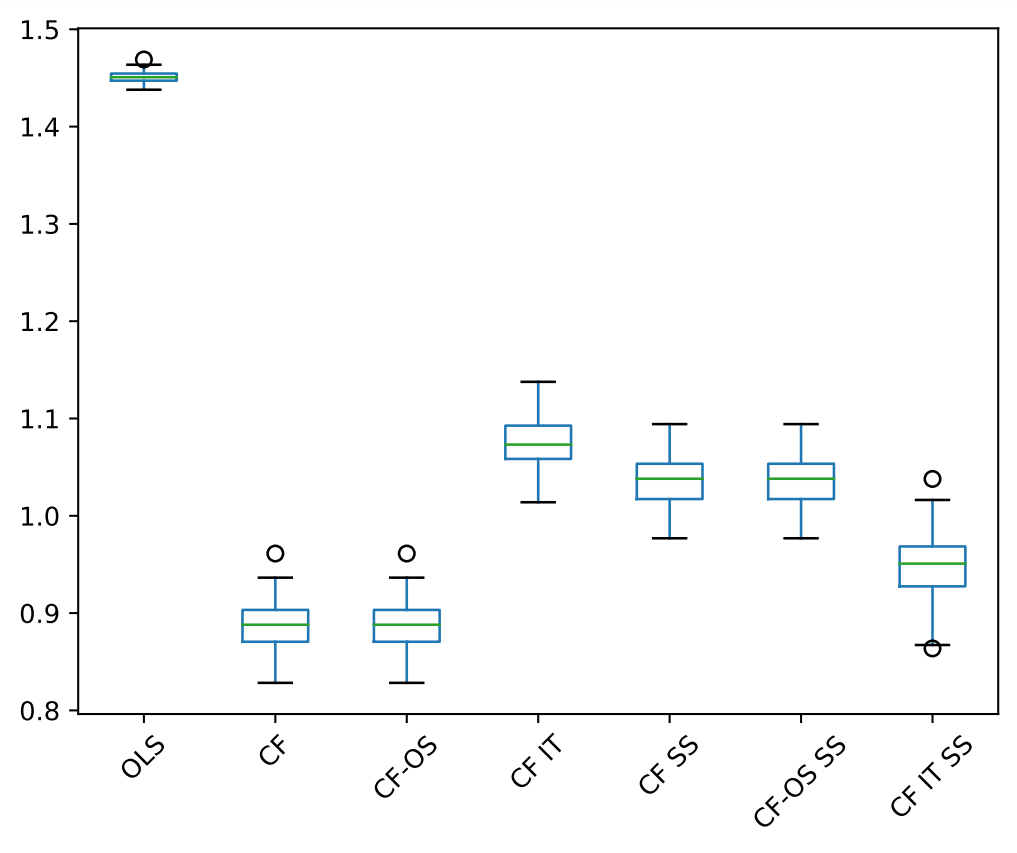
with: $h(\tilde{x}_{it}) = sin(2 * \tilde{x}_{it}) + exp(-16 * \tilde{x}_{it}^2)$,
$g(\tilde{x}_{it}, z_{it}) = log(0.1 * \tilde{x}_{it}^2 + z_{it}^2)$.

$$\tilde{x}_{it} = \alpha_i/2 + \zeta_{it}, \quad with \quad \zeta_{it} \sim U(-2,2), \tag{28}$$

$$z_{it} = \alpha_i/2 + \nu_{it}, \quad with \quad \nu_{it} \sim U(-2,2), \tag{29}$$

$u_{it} \sim U(-2,2)$, $\omega_{it} \sim U(-1,1)$, $\alpha_i \sim N(0,1)$.

Simulation results



(a) N=5000, T=3

Figure 2: Linear structural equation and nonlinear reduced-form equation: DGP 5-Scenario 1. Ivs: $\tilde{x}_{it}$, z_{it}, $\tilde{x}_{it-1}$, z_{it-1}

Note: The true parameter value is 1 ($\beta_1 = 1$).

Conclusion

1. Linear structural equation:
   1.1 Using a Monte Carlo experiment we conclude that CF-OS has lower bias and RMSE than 2SLS on the first-differenced model in almost all scenarios.
2. Partial linear structural equation:
   2.1 Using a Monte Carlo experiment we conclude that CF-IT with data splitting has lower bias and RMSE than 2SLS on the first-differenced model.

Thank you for your attention

Annex

Estimation: CF - OS

The estimation procedure has five steps:
Step 1 and **Step 2** equal to CF.

Step 3: Use the residuals estimate ρ using Frisch-Waugh-Lovell theorem.

Step 4: Obtain $\Delta\tilde{v}_{it} = \widehat{\Delta v_{it}} - \widehat{\widetilde{\Delta u_{it}}}\hat{\rho}$

Step 5: Use the new residuals to estimate the parameter of interest.

We call this estimator CF-OS in the simulation study.

Estimation: CF - OS

The estimation procedure has five steps:
Step 1 and **Step 2** equal to CF.

Step 3: Use the residuals estimate ρ using Frisch-Waugh-Lovell theorem.

Step 4: Obtain $\Delta\tilde{v}_{it} = \widehat{\Delta v_{it}} - \widehat{\widetilde{\Delta u_{it}}}\hat{\rho}$

Step 5: Use the new residuals to estimate the parameter of interest.

We call this estimator CF-OS in the simulation study.

Limitations and Work in progress

1. Low-dimensional model: extend simulation with high-dimensional covariates.
2. MA(1) errors after FD: use of improvements of RF such as RF-GLS (Saha et al, JASA 2023), and MERF (Hajjem et al,JSCS 2012) with Mundlak approach.
3. No theoretical guarantees: statistical properties of the estimators.